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Toughness: Determination and Definition [and Discussion]

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FRACTURE MECHANICS REVIEW

Toughness: determination and definition

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The aim of using the fracture mechanics approach to fracture control in engineering structures is to determine the loading (applied, thermal, residual stress, etc.) at which a pre-existing crack of given size will extend in the most vulnerable mode (brittle, ductile tearing, fatigue, stress corrosion, high-temperature creep). *The means* involves:

(1) the determination of loading conditions for crack extension in a sharply notched specimen by using the appropriate material and environmental condition (temperature, loading rate, gaseous and electrochemical surface condition, pressure);

(2) provision of a stress analysis embracing the sharp notch in both distribution and *scale*, and the effects of shape and loading conditions.

The paper is concerned with these basic aspects under circumstances restricted to elastic behaviour modified only by local yielding.

Objectives

Many attempts were made during the first half of this century to devise a means of description of the processes of fracture that could be used to predict the behaviour of structural components from that of simple laboratory tests, not without some successes. Much significance must be accorded to the development of impact testing of small notched bars, since this gave rise to the well known transition temperature approach for structural steels, and this approach still has much to commend it in terms of simplicity, economy of testing and use for purposes of quality control. The use of impact testing, nevertheless, has always relied upon empirical correlations with full-scale behaviour, so that its limitations become more obvious as structures become larger, and the full-scale testing of their components is correspondingly less practicable.

The subject area now clearly recognized as fracture mechanics grew inexorably out of the need for a more catholic means of description of the fracture process, and it may be reduced in essence to definition and equation of those conditions of driving force and resistance that lead to the extension of cracks. Almost countless worldwide contributions have now been made to this particular study, but few would deny the priority to be extended to the sustained vision of George Irwin, who now for thirty-five years has consistently nourished a belief that a workable hypothetical framework can be found as outlined in the synopsis of this paper (Sih *et al.* 1975). Irwin, more than almost any other, understood and patiently promulgated the idea of a fracture analysis plan, with meticulous attention to definitions and nomenclature, so that evidence from measurement from all sources might be combined, matched and criticized, to create a common language of fracture that can be widely comprehended and applied. After two nominal retirements from formal appointments, Professor Irwin continues to lead an effective team at University of Maryland whose studies of fast fracture in photoelastic materials are breaking new ground in characterizing dynamic crack tip stress fields (Rossmanith & Irwin 1979).

Such is surely a good example of the application of scientific method, but it can still be abused by blind acceptance. Using the analogy of structural mechanics, the framework of fracture mechanics continues to contain a redundancy of members and will continue to do so into the foreseeable future of its development, leading to local conditions of incompatibility and internal strain. Much remains to be done to improve the perfection of its form, but it is already a strong framework, and suitable to be used and leaned upon in the most practical way. This is particularly true with regard to linear elastic fracture mechanics (l.e.f.m.); the methods of elastic–plastic fracture mechanics are as yet more empirical, but it may be claimed that they too lead to safe methods of fracture control, even if the margins are not yet as uniform or consistent.

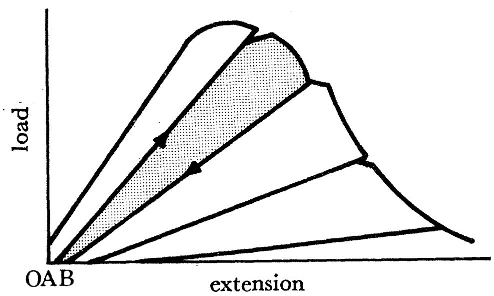

Figure 1. Stable, repeatedly incremental crack extension of a precracked specimen of a tough material.

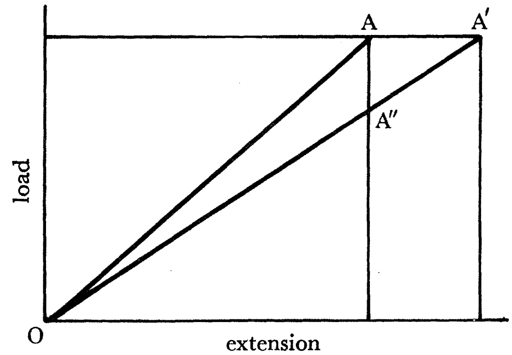

Figure 2. The effect of incremental crack extension on unloading compliance (l.e.f.m.).

The size effect

Notwithstanding the terminology and treatment pioneered by Irwin, it is useful to consider the method of fracture toughness measurement devised quite independently by Charles Gurney (Gurney & Mai 1974) for a narrow range of moderately thin, tough materials, which minimizes all preconceptions and is conveniently conducted on a pin-loaded edge-precracked specimen (like the compact tension specimen) with slow loading and unloading in a testing machine provided with autographic recording of load and extension between the pins. The specimen is first loaded until there is an increment of crack extension which is measured, and it is then unloaded. The process is continued until the stable crack has severed the specimen. The cumulative load-extension curve resembles figure 1, from which it is observed that unloading and reloading have substantial linear components, and the external work of fracture is contained within the envelope and its constituents. Comparison of the shaded component areas in most cases that increments of fracture work are proportional to increments of cracked area, and their corresponding increment of crack length confirms so that a ratio may be derived, which is defined as the fracture toughness. However, it will also be observed that the unloading lines are associated with increments of permanent set, and the detailed stress analysis of a specimen tested by this method is complex. The method does not have wide application because unstable fracture intervenes at an early stage with most structural materials that are to be controlled under economic conditions of use.

The existence of a uniform value of fracture work per unit crack travel (constant fracture toughness) implies the corresponding existence of a size effect; if the specimen size is substantially increased in geometric proportion, the behaviour becomes unstable at an earlier stage, and if it is decreased the observation of a regular crack extension is obscured by large and widespread prior deformations. In spite of this, the Gurney test reveals an important characteristic, that of the change of compliance with crack extension, and in most practical cases where unstable or brittle fracture intervenes, the compliance is an elastic property, calculable by stress analysis as the reciprocal of stiffness.

Compliance

With restriction to linear-elastic conditions, the loading–unloading curve for a small increment of crack extension becomes as in figure 2. At constant load during this extension the line moves from OA to OA′; at constant extension (fixed grips) it moves from OA to OA″. For load P per unit thickness, the specimen extension is PC, where C is the compliance, and the associated external work equivalent to elastic stored energy is $\frac{1}{2}P^2C$. If crack length is a, and G is strain energy release rate (or crack-extension force),

$$G = \tfrac{1}{2}P^2\, \partial C/\partial a \qquad (1)$$

in the limit of infinitesimal crack extension ∂a; under the same condition the area AA′A″ vanishes, and G is the same for fixed load or fixed grips.

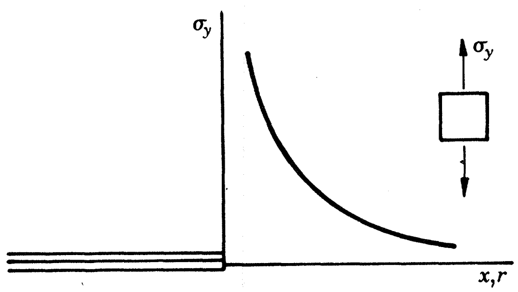

Figure 3. Concentrated stress at a sharp crack tip (l.e.f.m.) (equation (2)).

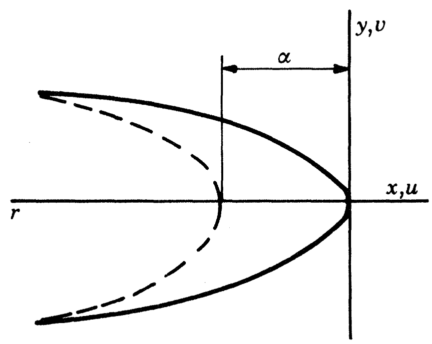

Figure 4. The local elastic displacement field associated with small crack extension (l.e.f.m.) (equation (3)).

Linear-elastic crack tip stresses and displacements

Although this is a topic to which an immense amount of attention has been devoted, that which is mostly significant with regard to the cracking plane is contained in figures 3 and 4. From the first of these it is made evident that the major component of the stress across this plane at small distance r from an infinitely sharp crack tip is given by

$$\sigma_y = K/\surd(2\pi r), \qquad (2)$$

where K is a scaling constant known as stress intensity. Its units are now recognized as MPa $m^{\frac{1}{2}}$, but much valuable past recorded data can be recovered by noting that

$$10^3(\text{lbf in}^{-2})\text{in}^{\frac{1}{2}} = 6.90\ \text{MPa} \times (0.025\ \text{m})^{\frac{1}{2}} = 1.10\ \text{MPa m}^{\frac{1}{2}}.$$

In figure 4 it is shown that the corresponding elastic crack opening displacement v at a corresponding small distance r from the sharp crack tip (now measured in the reverse direction) is

$$v = \frac{4K}{E}\sqrt{\frac{r}{2\pi}}. \qquad (3)$$

By closing up an incremental short length α of such a sharp crack, Irwin showed that the work $G\alpha$ done by the crack closure forces would be $K^2\alpha/E$, so that the crack extension force G is related to the stress intensity K by

$$G = K^2/E. \tag{4}$$

These are the conditions for a thin plate in plane stress; for the alternative condition of plane strain it is adequate to substitute the corresponding elastic modulus $E' = E/(1-\nu^2)$, where ν is the Poisson ratio.

K is much more convenient to apply than G as a stress field scaling parameter, since simple algebraic summations may be performed for multiple loadings instead of the alternative

$$G_{\text{sum}} = [\sqrt{G_1}+\sqrt{G_2}+\sqrt{G_3}\,\ldots\ldots]^2,$$

where components are ambiguous with respect to signs. Hence, K is the universal published output from linear-elastic stress analyses of loaded, precracked plates and shapes, although the compliance method may be conveniently applied where finite element calculations are performed. It is usually sufficient to have about ten elements within the crack length, but results are always overestimated by finite element methods, compared with results from more rigorous analytical methods that fully account for the crack tip stress infinity, or singularity. Reference may nowadays be made to excellent and voluminous compendia of solutions for hypothetical cracks of given lengths and orientations in bodies with a wide variety of shapes and loading modes.

Nomenclature

K and G, being stress intensity and crack extension force respectively, are measures of crack tip loading. They have critical values at crack extension, K_c and G_c, known in both cases as fracture toughness. Suffixes 1, 2 and 3 are used to designate modes as follows: 1, in-plane loading; 2, shear parallel to crack direction; 3, shear parallel to crack front. The latter occurs n the tearing of thin sheets and foils, and in torsion of shafts with longitudinal surface cracking as at keyways. Mixed mode crack extension can occur in rarely encountered cases.

Standard specimens

The ubiquity of the linear-elastic fracture mechanics approach is demonstrated by the range of alternative specimen shapes and loading configurations which may be employed in the measurement of fracture toughness, for instance with mode 1 loading. These may have internal or external notches, and symmetric or asymmetric loading. Nevertheless, the preference is often narrowed to plane single edge notched configurations with bending or eccentric tension loading, and the so-called compact tension specimen has many advantages. Some of the restrictions are mentioned below. It is usually necessary to employ an autographic clip gauge clamped across the crack flanks to detect the onset of loading nonlinearity.

Measurement capacity

Although there will always be at least a small plastic zone at the crack tip due to stress concentration within the elastic region having an inverse square-root characteristic stress distribution, even at the smallest applied loads, unacceptable errors are obtained with linear-elastic treatments when this zone spreads to embrace a large proportional volume of the

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specimen. The degree of nonlinearity of the load–extension curve for the specimen provides a suitable criterion for the validity of the result, when linear-elastic factors are used to estimate K_c values at incipient crack extension arising from measured loads. Each specimen of given size then has a maximum measurement capacity, which may be coarsely estimated in terms of the load to cause general yielding. A larger geometrically similar specimen must be used for retest if an invalid result is obtained. One test for validity, conducted after the initial result for K_c has been obtained, is to ensure that

$$(K_c/\sigma_Y)^2 \leqslant b, \tag{5}$$

where σ_Y is uniaxial yield stress, and b is a representative length, being the smaller of crack depth or ligament length, or plate thickness in addition in the circumstances described below

Plane strain fracture toughness

When yielding occurs at the crack tip, the condition of constant volume during deformation under conditions of biaxial tension causes dimpling, or local thickness reduction. This effect is constrained when the specimen is thick so that triaxial tension is induced. The lower limiting fracture toughness conditions are usually obtained under these circumstances, which are defined as plane strain. To obtain a valid plane strain fracture toughness K_{Ic}, the reference length must be considered to be limited to 0.4 times that defined above.

Whereas valid plane strain fracture toughness measures may be obtained from specimens of high strength metals, and solids thought of as brittle, by using testing machines of relatively small standard laboratory load capacities, very large testing machines are required for soft metals and even mild steels. A useful rule of thumb is that if a valid K_{Ic} can be measured for a material on a standard laboratory testing machine, the material is too brittle for rugged engineering applications!

Notch sharpness

Precise measurements of K_{Ic} also require the use of sharp crack tips, which are normally fatigue sharpened machined notches of small radius. The size of crack tip yield zone produced by the fatigue loading must be much less than that induced by application of the K_{Ic} measurement load. The full validity criteria are set out in A.S.T.M. Testing Standard E399, and at length, with commentary, by Knott (1973).

Extensive yielding

Invalid fracture toughness values represent artefacts of the stress analysis and associated testing method, and not of the materials being assessed. It is necessary to encompass yielding in the stress analysis to provide a satisfactory predictive treatment for tough materials. In general, this is more necessary in relation to the standard fracture toughness specimens than with regard to the structures for fracture control and assessment.

Plastic zone crack length corrections (figure 5)

Since
$$r_Y = \frac{1}{2\pi}\left(\frac{K}{\sigma_Y}\right)^2$$

and the plastic zone must extend to O′, so that the area O′A″BC equals that of OAA′BC, if the same load is to be sustained as in a wholly elastic specimen, it follows that the effective elastic length of a crack with local yielding is greater by r_Y. This is the elementary plastic zone crack length correction.

A conceptual estimate of the crack opening displacement $\delta = 2v$, making use of r_Y and (3), leads to the result
$$\delta = \frac{4K^2}{\pi\sigma_Y E} = \frac{4G}{\pi\sigma_Y}. \qquad (6)$$

However, this is an approximation, and an alternative expression derived from the work of incremental crack extension is preferred. Thus,
$$G = \sigma_Y \delta, \quad \text{or} \quad \delta = G/\sigma_Y.$$

σ_Y may contain a factor $m \geqslant 1$, making allowance for an intensification due to multiaxial yielding.

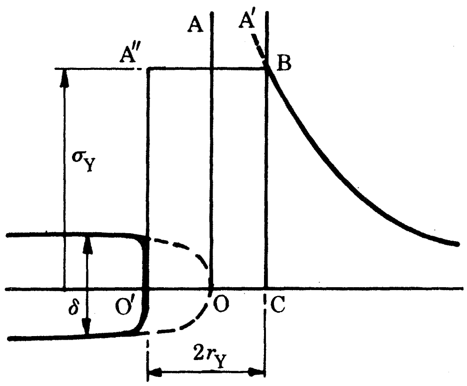

Figure 5. Local yielding at a crack tip and the plastic zone crack length correction.

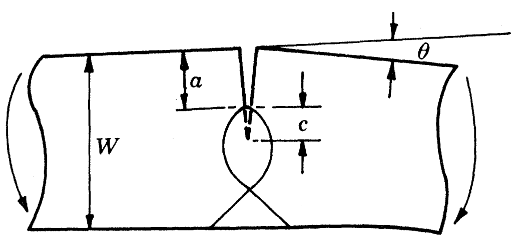

Figure 6. Plastic hingeing and the development of crack opening displacement in a notched bend specimen.

Crack opening displacement

Having identified a conceptual crack opening displacement, uniquely related to K, G and σ_Y for small-scale yielding, it may be observed

(1) that real cracks in yielding materials become blunted and flat-ended when strained open, and

(2) that calculated plane strain slip line fields and counterparts exposed by Fry's etchant in such as the Charpy V-notch specimen in bending expose a hinge action leading to a large increment of crack opening displacement when general yield has occurred. Then $\delta = c\theta$ in figure 6.

Since δ or c.o.d. has observed significance over the whole range of loading, whereby it increases by several decades of magnitude as straining increases, and since it may be identified with K and G when there is only small-scale yielding, the same relations may be adopted over the whole range, to calculate equivalent values of K and G for large-scale yielding when there are no alternatives.

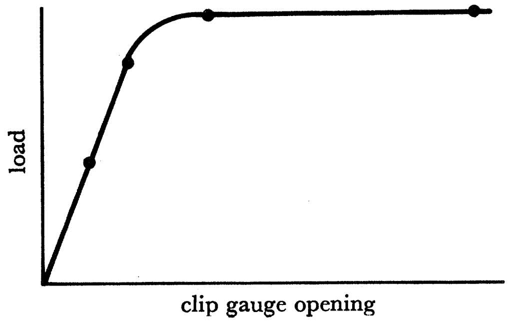

FIGURE 7. Development of nonlinearity due to plasticity in a notched bend specimen.

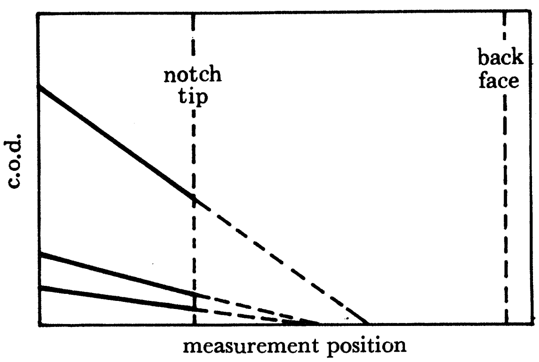

FIGURE 8. Displacements at the flanks of a crack as they are observed in a notched bend specimen progressively loaded to yield.

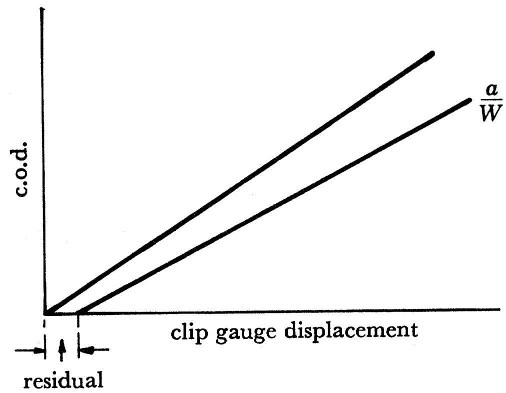

FIGURE 9. The inference of crack tip opening displacement from clip gauge measurements on a notched bend specimen.

VERIFICATION (figure 7)

T. C. Harrison conducted notch bend tests on pipeline steel and measured opening displacements along the whole length of the notch flank, using microhardness indentations with accurate photography (figure 8). Hingeing action was confirmed, with the instantaneous centre of rotation moving from the notch tip to 0.45 of ligament depth at full yield. The relation between crack opening displacement and clip gauge displacement is parabolic in the elastic loading state, and incrementally linear thereafter. A residual clip gauge displacement may be defined to describe this behaviour (figure 9), which is formulated in BS 5762, from the rationalized results of several independent finite element (f.e.) elastic–plastic analyses for materials with zero or small strain hardening. Such f.e. correlations are now available for the principal configurations of specimens in bending and tension.

Although the c.o.d. has a simple concept, and is readily measured or inferred from clip gauge observations, it has two main disadvantages:

(1) it loses definition at small applied loads, where the crack tip profile is essentially parabolic;

(2) its application to correlation between fracture toughness specimens and structural components is confined to materials with small strain hardening.

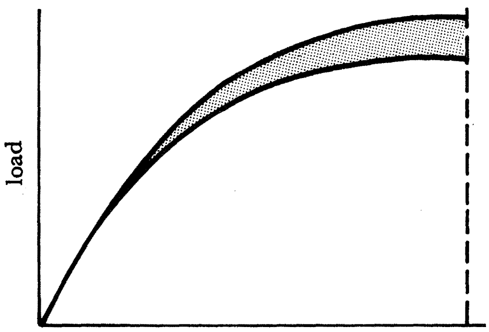

FIGURE 10. Incremental external work associated with small crack extension in a nonlinear elastic material.

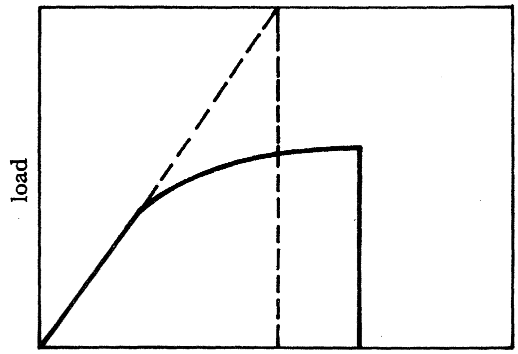

FIGURE 11. Equivalent linear elastic behaviour with respect to external work, as inferred in the 'equivalent energy' treatment of plasticity effects.

THE J-INTEGRAL

This alternative fracture criterion, analogous to crack extension force G, may be defined for any linear or nonlinear stress–strain relation, with the reservation that it be elastic (i.e. reversible). It takes the form

$$J = \int_{\Gamma} (W\,\mathrm{d}y - T\,\partial u/\partial x)\,\mathrm{d}s, \tag{7}$$

where W is strain energy volumetric density (a nonlinear elastic material is assumed), T is the traction vector exterior to any path Γ around the crack tip from one notch flank to the other, u is displacement in the direction of T, and s is arc length along the integration path.

With real elastic–plastic materials the nonlinear elastic assumption is invalidated as soon as crack extension occurs, and stresses change direction and sign around the crack tip. Exhaustive f.e. elastic–plastic computations demonstrate that the error from this source is not large.

J is related conceptually both with incremental (figure 10) and total strain energy, and has led in the latter case to an equivalent energy concept acceptable for bend specimens only, in which the specimen is treated as though it had undergone a linear elastic loading equivalent to the area shown dotted in figure 11.

A valuable comparison, which leads to the conclusion that J and c.o.d. are linearly related through yield stress σ_Y (triaxially supplemented, where necessary), arises in the particular case of fully plastic, zero strain hardening incremental loading.

Relations between J and c.o.d.

A correlation may be made for the particular case of fully plastic, zero strain hardening incremental loading of a rectangular specimen such as the compact tension specimen (figure 12), in which it is known that the deformation takes place by plastic hingeing across the minimum section, with rotation about a hinge point located in this ligament. The material remains linear-elastic, at constant stress and strain, outside the plastic zone. Load is P and incremental rotation is θ. The integration path is taken along the rectangular boundary of the specimen. There is no contribution to J from the two faces AB, since $T = 0$ and W is invariant; or from the first term along BC, since y is constant. Assuming that the forces P are not quite concentrated to points, so that

$$T\,\mathrm{d}s = P \quad \text{and} \quad \partial u/\partial x = \tfrac{1}{2}\theta,$$

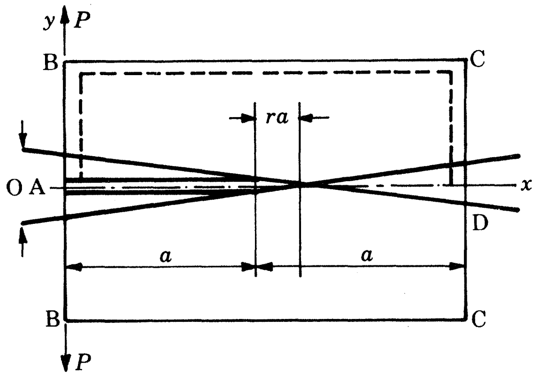

Figure 12. Incremental J-integral summation for fully plastic components of deformation in a compact tension specimen of characteristic width $W = 2a$.

the contribution to J from the second term along BC is $P\theta$. There is no contribution from the second term along sides CD, since $T = 0$, so that the remnant from the second term is confined to incremental plastic deformation, amounting to $(1-r)a\theta$, where r defines the plastic hinge position as in figure 12. The total increment of J is therefore $\theta[P+(1-r)a\sigma_Y]$. P and r may be computed from a slip-line analysis. The result may be expressed in terms of JW/U, where W is gross width and U is incremental work, given by $P(1+r)a\theta$. Comparison may be made with a linear-elastic and a f.e. elastic–plastic determination, as follows.

	incremental fully plastic	f.e. elastic–plastic	linear-elastic
JW/U	4.72	4.88	4.96

A more general analysis for an arbitrary stress–strain curve leads to the ratio 4, and the difference may be due to the tension component, since the general analysis is for pure bending. This comparison draws attention to a feature that limits the application of the equivalent energy treatment noted above, which is that for other shapes of specimen the relation between J and U is usually much more bilinear than tabulated here.

Resistance curves

C.o.d. loses definition at first crack extension, because the crack front loses its blunted form. J loses definition under the same condition because reversibility of deformation is lost. Nevertheless, there is much interest in the growth of resistance with crack extension, which may be measured assuming in the computation of δ or J that negligible crack extension has taken place. Curves of δ or J against initial slow crack extension are known as R curves, and nearly always have a continuously rising form, until instability occurs.

Discussion

The foregoing represents a concentrated review of methods of definition of fracture criteria, and it is wise to distinguish yet again on the one hand the already refined character of linear-elastic treatments, and on the other hand the potential for further development that exists for elastic–plastic treatments. It should also be recognized that the latter are representative of a far more difficult problem. It was pointed out by McClintock (1965) that it is unlikely that a one-parameter representation of elastic–plastic fracture toughness could be totally successful, in view of the additional variables that must be accommodated in the consideration of plasticity with multiaxial stresses. Even with linear-elastic fracture mechanics it was empirically determined long ago that valid measurements of minimum toughness required the geometric limitations of (5). Similar geometric limitations have been discovered for elastic–plastic treatments (Wells 1971), but the added disadvantage in this case is that the structural components in which cracks are required to be assessed and controlled also exhibit these effects. It is important, for instance, that relatively shallow cracks exhibit substantially larger toughness for otherwise equal conditions, than laboratory measurements would indicate. Moreover, it is not practicable at present to interpolate quantitatively between the ideals of plane stress and plane strain in finite element elastic–plastic analyses relating to finite plate thicknesses. So the treatments remain conservative, to contain these effects, and the conservatism is unpalatable to those concerned with the economics of large, real structures.

The development of fracture mechanics has always thrived in an atmosphere of scepticism and criticism; it will continue to be improved thereby.

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Discussion

D. A. Chamberlain (*Department of Civil Engineering, The City University, London EC1V 0HB, U.K.*). It is interesting that some practitioners regard a switch between the plane stress and plane strain conditions, to be simply a matter of applying a factor $(1-\nu^2)$, where ν is the Poisson ratio. While plane strain conditions may prevail at the inner region of a planar fracture specimen, it is difficult to accept a physical application of the plane stress condition. Where a crack breaks out into a free surface, the stress state is known to be extremely complex, and far removed from plane stress. For a through-thickness crack in a relatively thin planar specimen, it may be worthwhile to isolate surface boundary layers, as distinct from the inner plane strain region. The relative thickness proportions of these layers may account for some of the thickness dependence that is often a problem to practitioners. I should greatly appreciate any clarification on the justification of the stress state switch.

P. Hancock (*Department of Materials, Cranfield Institute of Technology, U.K.*). The question has been raised concerning the requirement for accurate elastic/plastic solutions for cracks in structural components. As the scatter in c.o.d. and J data measured under carefully controlled laboratory conditions is still very considerable, particularly in welded test specimens, I should therefore like to ask how rigorously do we require accurate analytical elastic plastic solutions when there is such a considerable scatter with these baseline data?

A. A. Wells. Mr Chamberlain draws attention to a confusion that may exist concerning differences between the plane strain and plane stress modes. When the idealized conditions are wholly elastic with given loading in a plane body containing a crack, the in-plane stresses are independent of through-thickness constraint, but the strains and displacements are reduced by the factor $(1-\nu^2)$ in passing from plane stress to plane strain. This change is not great. However, all materials of engineering utility show some plasticity, even when they are regarded as brittle, and the elastic stresses in an outer field surrounding the sharp notch are then governed by conditions at the elastic–plastic boundary. The difference between plane strain and stress conditions within the notch tip plastic zone is substantial and is summarized, for instance, by Knott (1973, ch. 3). Maximum stress intensifications may differ in the two extreme cases by a factor of about 2 or 3, so that alternative fracture modes may be involved. It was recognized in this respect by Irwin and others well before 1965 (A.S.T.M. 1965) from the results of experiments with cracked planar specimens that the most important variable is the ratio of the yield zone size to section thickness. This ratio may be conveniently quantified, making use as comparators of the uniaxial yield stress on the one hand, and a purely elastic reference stress assumed to govern the position of the elastic–plastic boundary, on the other. It matters little in identifying the onset with increased loading (before crack extension) of the transition from plane strain to plane stress that this calculation is formalized, and not an exact description of the complex stress state, provided that consistency is maintained, since the change of fracture mode frequently involves an almost stepwise change of observed fracture toughness.

However, there are secondary variables, also as recognized by Mr Chamberlain. Two of the more important of these involve the ratio of the plastic zone size (as formally calculated) to the ligament length behind the crack, or the length to the surface in the case of an edge crack. The conditions recognized in linear elastic fracture mechanics demand that neither of these two lengths shall be less than section thickness if a measurement of plane strain fracture toughness is to be validated. Similar limitations were observed by this author with respect to measurements of critical crack opening displacements (Wells 1971), in a wide plate test series with edge cracks that embraced both small and large lengths compared with section thickness. A minimum c.o.d. for fracture initiation is experienced for given material, test temperature and strain rate when an edge crack in a wide specimen has a depth equal to the section thickness. Much larger values arise for shallower cracks, and a difficulty is thereby created in that the c.o.d. approach to fracture control under elastic–plastic conditions has been observed in some cases to be conservative, since crack-like defects in practical structures are also often shallow, and at the surface.

It is recommended, when such circumstances can occur, that not only c.o.d. should specimens employ the full section thickness of the structure, but that at least two alternative shapes of specimen should be considered. The first of these is that recognized for l.e.f.m. measurements of plane strain (otherwise considered to be a minimum) fracture toughness, and this specimen has a gross depth equal to twice the section thickness, with notching to half depth. The second, for measurements relating to surface cracks in structures, may be of half this gross depth, with about 30% notch depth. Such a specimen is appropriate to the heterogeneous local fracture conditions found at some fusion welded joints, and can be expected to correlate better in terms of transition temperature with fractures in notched and welded wide plates.

The justification sought by Mr Chamberlain for the existence of the stress state transition with elastic–plastic fracture is therefore an experimental one at present, in almost complete conformity with the limitations found to be applicable with l.e.f.m. The complete theoretical elucidation of complex stress states, as stated in the paper, remains largely as a future task, since it involves three-dimensional yielding conditions at sharp notches.

Professor Hancock questions whether these accurate elastic–plastic solutions are really needed, in the light of the recognized large scatter of fracture toughness measurements when an elastic–plastic method such as J or c.o.d. is used to characterize fracture toughness. He will probably accept that the scatter problem is mainly experienced at temperatures near to the fracture mode transition. The argument is that an important aspect of the scatter under these conditions is traceable to small variations of stress intensification having considerable leverage in terms of transition temperature; some of these small variations may be due to yield point elevation and variation in weld metal due in turn to minor differences in the manner of deposit. This has been suggested in other discussion at this symposium. However, there are the other sources of variation of stress intensification that experiment has suggested are due to specimen geometric effects, and notably those due to relative notch depth. The value of the accurate elastic–plastic solutions would be to shed light on these variations of stress intensification and so to enable the developers of materials, and notably those concerned with welding consumables and procedures, to distinguish geometric and metallurgical effects. Finally, Professor Hancock seeks an opinion on the degree of precision which is being sought. The comparisons presented in my paper at the last such Royal Society Discussion Meeting on fracture in structures (Wells 1965) suggested that differences in stress intensification of $\pm 5\%$ are quite important, and the considerable acquisitions of evidence from finite element solutions that have since appeared do not change that view.

References

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